Book of abstracts

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Faculty of Mathematics and Computer Science
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Ibrahim Alraddadi
Islamic University of Madinah

The Asymmetric Periodically Forced Van Der Pol Oscillator

We review geometric singular perturbation theory (GSPT) which has been used to explain the behaviour of the singular slow-fast system near the singular limit. In particular, we follow the analysis of Guckenheimer et al. [10] for the periodically forced symmetric van der Pol oscillator ($\beta = 0$), then we constructed the Poincare return map for studying the bifurcation phenomena of this model. We generalise to a asymmetric forced van der Pol oscillator for $\beta \neq 0$. We show that the forced asymmetric van der Pol oscillator can become frequency locked due to the forcing. Then, we extend this analysis to show how the symmetry breaking parameter β in a periodically forced van der Pol oscillator influences the width of Arnold tongues (also known as frequency locking regions), and we find these frequency locking regions in the parameter space (α, ω) [1] The Asymmetric Periodically Forced Van Der Pol Oscillator. (2025). European Journal of Pure and Applied Mathematics, 18(1), 5787.

Gianni Arioli
Politecnico di Milano

A comparative study of validated Taylor and Chebyshev integration of ODEs

Recent advances in the study of dynamical systems have increasingly relied on the rigorous numerical integration of finite-dimensional ordinary differential equations (ODEs). There exist different approaches to this problem, with the Taylor expansion of solutions representing the most traditional one. However, in recent years, the Chebyshev expansion has attracted a lot of attention, primarily due to its ability to allow substantially longer integration steps in many cases. This paper undertakes a comprehensive analysis of these two methods, evaluating their performance and efficacy when applied to a range of representative systems, including the logistic equation, Newton's equation with various potential functions, and the Lorenz system.

Bogdan Batko
Jagiellonian University

Combinatorial approach to sampled dynamics based on Gaussian process modeling

We propose a novel method combining combinatorial topological dynamics and Gaussian process (GP) modeling, whereby we can characterize the global dynamics from a finite amount of data, with high confidence. More specifically, the data is used to construct a surrogate GP model. We then describe the dynamics using algebraic topological invariants inferred from a combinatorial multivector field built on the basis of the GP model. Our experiments show that relatively sparse data is sufficient to obtain a qualitative description of the underlying dynamics with high confidence.

Omer Bobrowski
Queen Mary University of London

Universality in Random Topology

Random geometric complexes are simplicial complexes (high-dimensional graphs) whose vertices are generated by a random point process in a metric space. In this talk we will focus on the homology (cycles/holes in various dimensions) of these complexes. Our main results show that the lifetime distribution of homological cycles obeys a universal law, that depends on neither the support nor the original distribution of the point process. We will focus on the notion of "weak universality", addressing Poisson or binomial processes. We will present the main universality statement and the key steps for proving it. In fact, we will show that this notion of universality applies in a much broader context to scale-invariant geometric functionals (for example, the degree distribution in the k-NN graph). In addition, we will discuss "strong universality", which applies for a much wider class of point-cloud distributions, and is currently an open conjecture.

Matthieu Cadiot
McGill University

Spectral stability of localized solutions

In this talk, I will present a computer-assisted approach for investigating spectral stability of localized solutions, that is solutions vanishing at infinity, in PDEs and nonlocal equations on $\mathbb{R}^m$. In particular, the method is applicable to the study of stability of stationary localized patterns and solitary (traveling) waves. Applications to the planar Swift-Hohenberg PDE, to the capillary-gravity Whitham equation and to the Gray-Scott model will be exposed.

Maciej Capiński
AGH University of Kraków

Arnold Diffusion in the Full Three-Body Problem

We show the existence of Arnold diffusion in the planar full three-body problem, which is expressed as a perturbation of a Kepler problem and a planar circular restricted three-body problem, with the perturbation parameter being the mass of the smallest body. In this context, we obtain Arnold diffusion in terms of a transfer of energy, in an amount independent of the perturbation parameter, between the Kepler problem and the restricted three-body problem. Our argument is based on a topological method based on correctly aligned windows which is implemented into a computer assisted proof. This approach can be applied to physically relevant masses of the bodies, such as those in a Neptune-Triton-asteroid system. In this case, we obtain explicit estimates for the range of the perturbation parameter and for the diffusion time.

Chiara Caracciolo
University of Padua

Existence of KAM tori in the realistic planar three-body problem

We present a computer-assisted proof of the existence of KAM tori in the planar planetary three-body problem, using physically realistic parameters. The proof is implemented through publicly available software (Locatelli 2023, Mendeley Data) and is based on the construction of the Kolmogorov normal form. We apply this proof to Hamiltonians of the planar planetary three-body problem, after a few preliminary changes of coordinates. The procedure has been tested for different values of the planetary and stellar masses, as well as orbital parameters (semi-major axes and eccentricities), all taken from real observational data of two-planet exoplanetary systems. The proof succeeds for the systems HD11964, HD142, and HD4732, all of which present non-small orbital eccentricities. Finally, we propose a simple criterion to identify planetary systems for which this approach is more likely to succeed.

This is a joint work with Ugo Locatelli (University of Rome "Tor Vergata").

Wojciech Chachólski
KTH, Stockholm

Data, geometry and homology

For a successful analysis a suitable representation of data by objects amenable for statistical methods is fundamental. There has been an explosion of applications in which homological representations of data played a significant role. I will present one such representation called stable rank and introduce various novel ways of using it to encode geometry, and then analyse, data. I will provide several illustrative examples of how to use stable ranks to find meaningful results.

Pieter Collins
Maastricht University

The Ariadne Framework for Rigorous Numerics

In this talk I will give an introduction to the conceptual framework of the Ariadne tool (`www.ariadne-cps.org`) for formal verification of cyber-physical systems. The tool performs reachability analysison nonlinear hybrid automata and based on a general-purpose library for rigorous numerics, and is written in C++ with a Python interface for easy scripting. The tool has a well-defined semantics based on the theory of computable analysis, and objects are classified both by the mathematical type they represent (numbers, functions, sets), by the kind of information they provide (exact, effective, validated or approximate). Concrete data types are used for efficient computation, and polymorphic data types and solvers provide a uniform and extensible interface to data and algorithms.

I will end the talk by discussing on two future directions which I believe will be crucial in the further development of rigorous numerics. For the first direction, "verified verification", the goal is to increase trust in the tooling, by verifying the code an algorithms used are themselves correct, which can be done within a theorem proving environment such as Rocq (formerly Coq) or Lean. For the second direction, we should aim to bring rigorous numerics into the mainstream of scientific computing by developing software which is simple and intuitive to use for undergraduate students and non-specialists, but also powerful enough for cutting edge research applications in mathematics, science and engineering.

Jakub Czwórnóg
AGH University of Krakow

Continuation and bifurcations of periodic orbits and symbolic dynamics in the Swift–Hohenberg equation

I will present a computer-assisted proof of the existence of two smooth branches of even periodic orbits in the stationary Swift-Hohenberg equation at the energy level $E = 0$. These periodic orbits satisfy certain geometric properties, which imply that the system has positive topological entropy for an explicit range of parameter values of the system. Moreover, these two branches are connected via saddle node bifurcation. The proof uses rigorous computation of bounds on certain Poincaré map and its higher order derivaties.

Marisa dos Reis Cantarino
Monash University

Blenders and robust transitivity for a family of derived-from-Anosov maps

Blenders originally emerged as objects in dynamical systems as an example given by C. Bonatti and L. Díaz of a system which is not uniformly hyperbolic but it is robustly transitive. Roughly speaking, on an n-manifold, a blender is a hyperbolic invariant subset of the system that allows for robust intersections of s-dimensional stable manifolds and u-dimensional unstable manifolds, with $s + u < n$. The way to make this intersection robust without "the right dimensions" is to make the stable set of the blender to "fill the space as if it is higher dimensional". This intersection allows the existence of robust heterodimensional cycles, giving conditions for robust transitivity. We present a family of D.A. (derived-from-Anosov) systems on the 3-torus for which we prove robust transitivity using the presence of blenders. These proofs use computer assisted strategies. This is ongoing work made with the collaboration of Andy Hammerlindl and Warwick Tucker.

Parker Duncan
Queen Mary University

Maximal Cycles in the Čech Complex

We study maximally persistent k-cycles for the Čech filtration built on a Poisson Point Process with respect to sufficiently nice density functions; maximal means that the cycle has the largest death-to-birth ratio. We show that these cycles scale, with high probability, like a constant multiple of $\log(n)/\log(\log(n))]^{1/k}$, and, most importantly, this scaling is independent of the density function. In other words, the result is universal, in some sense. The constant multiple in the limit is related to the covering number of the unit k-sphere.

Jordi-Lluís Figueras
Uppsala University

Self-Similar Singular Solutions to the Nonlinear Schrödinger and the Complex Ginzburg-Landau Equations

We prove the existence of radial self-similar singular solutions for the mass supercritical Nonlinear Schrödinger Equation far from the critical regime and, more generally, branches of such solutions for the Complex Ginzburg-Landau Equation. We are also able to control their monotone index (number of monotone intervals). In particular, we prove the existence of monotone radial self-similar singular solutions for the three dimensional cubic Nonlinear Schrödinger Equation. The paper combines sharp analytic bounds of the self-similar profile at infinity with computer assisted bounds around zero and their matching at an intermediate value. (Joint work with Joel Dahne).

Bartosz Furmanek
Jagiellonian University

Topological simplification guided by the depth poset

Topological simplification is the process of reducing complexity of data via topological methods, while maintaining its essential features. Consider a *Morse function* f on a simplicial complex X (see [1]). Recall that a *Morse complex* is a chain complex generated by critical cells of f and connections between them. Therefore, we can speak about persistence diagram of f, since f restricted to critical cells defines a filtration. The goal is to find another Morse function g on X which is close to f, but does not possess negligible birth-death pairs.

In case of 2-dimensional surfaces, as shown in [2], gradient of such g may be constructed by repeatedly reversing paths between critical cells of the gradient of f. Function g, constructed alongside, is L^∞-close to f and its persistence diagram does not possess birth-death pairs with small persistence, since reversing a (unique) path from t to s corresponds to moving the birth-death pair (s, t) to the diagonal.

In [3] it is shown that a persistence diagram admits a poset structure, the *depth poset*. Furthermore, its minimal elements, the *shallow pairs* (or apparent pairs) can be cancelled from the diagram, by algebraic operations defined on the complex, which preserve other birth-depth pairs.

However, to make use of this construction, we have to interpret those cancellations in the terms of the underlying complex, not its Morse complex. It turns out that, as in [2], they correspond to reversing paths between critical cells of f. Hence, for any ordering of pairs of critical cells (s_i, t_i) which agrees with the depth poset, we can find a morse function g, such that its persistence diagram is the one of f with pairs (s_i, t_i); provided there exists a unique path from t_i to s_i in X. The produced algorithm is dimension independent, with significant boost in the second dimension. This is work in progress with Jakub Leśkiewicz and Michał Lipiński.

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Bartosz Furmanek
Jagiellonian University

The existence of Conley complex with ring coefficients for gradient multivector fields

In [1], Forman presents an analogue of the classical Morse theory in the setting of a simplicial complex. A Forman vector field $\mathcal{V}$ is a partition of X into singletons (critical cells) and doubletons of adjacent dimensions (vectors). Under the assumption that $\mathcal{V}$ is gradient, Forman constructs a chain map $\Phi\colon C_*(X) \to C_*(X)$, an algebraic counterpart of the gradient flow and proves that its powers stabilize at a map Φ^∞, providing an isomorphism between the Morse complex of $\mathcal{V}$ and a subcomplex of Φ-invariant chains.

It is shown in [2] that in the setting of Forman gradient vector fields, this construction provides the existence of a Conley complex with arbitrary ring coefficients. However, this requires Forman vector field, excluding presence of general multivectors. The goal of the present research is to generalize the construction of Conley complex with arbitrary ring coefficients to the case when $\mathcal{V}$ is a gradient multivector field, where critical multivectors are singletons.

This is work in progress with Marian Mrozek and Thomas Wanner.

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[2] M. Mrozek, Th. Wanner. *Connection matrices in combinatorial topological dynamics.* SpringerBriefs in Mathematics, Springer-Verlag, 2025. ISBN 978-3-031-87599-1

Valery Gaiko
NAS of Belarus

Topological Methods for Studying Bifurcations and Multi-Stability of Polynomial Dynamical Systems

Using topological methods, we carry out a global bifurcation analysis of polynomial dynamical systems, study their multi-stability and applications. To control limit cycle bifurcations of such systems, especially, bifurcations of multiple limit cycles, it is necessary to know properties and combine effects of all their field rotation parameters. It can be done using the Wintner–Perko termination principle, according to which a maximal one-parameter family of multiple limit cycles terminates either at a singular point, which typically has the same multiplicity (cyclicity), or at separatrix cycle, which also typically has the same multiplicity (cyclicity). This principle is a consequence of the principle of natural termination which was stated for higher-dimensional dynamical systems by Wintner who studied one-parameter families of periodic orbits of the restricted three-body problem and proved that in the analytic case any one-parameter family of periodic orbits can be uniquely continued through any bifurcation except a period-doubling bifurcation. If we do not know the cyclicity of the termination points, then, applying canonical systems with field rotation parameters, we use geometric properties of the spirals filling the interior and exterior domains of limit cycles.

Applying the planar Wintner–Perko principle, we have solved, e. g., Hilbert's Sixteenth Problem on the maximum number and distribution of limit cycles for the general Liénard polynomial system with an arbitrary number of singular points, the Kukles cubic-linear system, the Euler–Lagrange–Liénard polynomial mechanical system, Leslie–Gower systems which model the population dynamics in real ecological or biomedical systems, and a reduced planar quartic Topp system which models the dynamics of diabetes. Finally, applying a similar approach, we have considered various applications of three-dimensional polynomial dynamical systems and, in particular, completed the strange attractor bifurcation scenario in Lorenz type systems connecting globally homoclinic, period-doubling, period-halving and Shilnikov type bifurcations of their limit cycles.

Zbigniew Galias
AGH University of Kraków

Is the classical Rössler attractor periodic?—a validated numerical study

The Rössler system is a classical low-dimensional dynamical system generating different types of attractors. The question whether the Rössler attractor observed for classical parameter values is periodic or chaotic remains an open problem. In this work, we search for periodic windows in a small neighborhood of the classical parameters with the hope to find a periodic window enclosing classical parameter values. Symbolic representation of trajectories is defined and the ordering of symbol sequences is used with a property that for short periodic symbol sequences their order agrees with the layout of corresponding periodic windows. Using symbolic descriptions of periodic window, the continuation technique, the bisection method, and the local exhaustive search, we find a periodic window with the distance smaller than $2 \cdot 10^{-22}$ from the classical case. Convergence properties of periodic attractors are studied numerically.

Marian Gidea
Yeshiva University

Geometric properties of normally hyperbolic invariant manifolds and scattering maps for conformally symplectic systems

Conformally symplectic systems appear naturally in physics (e.g., mechanical systems with dissipative force proportional to the velocity), celestial mechanics (e.g., the spin- orbit models), economics (e.g., discounted systems), transport problems (e.g., thermostats), etc. We will focus on conformally symplectic maps, which transforms the symplectic structure into a multiple of itself by a conformal factor.

First, we study geometric properties of normally hyperbolic manifolds(NHIMs) for conformally symplectic maps. We show that conditions among rates and the conformal factor are equivalent to the NHIM being symplectic. We also show that the hyperbolicity rates satisfy pairing rules similar to those for Lyapunov exponents of periodic orbits.

Second, we show that the scattering map – which relates the past asymptotic trajectory of any orbit in the homoclinic manifold to the future asymptotic trajectory – is symplectic. Joint work with R. de la Llave and T. M-Seara.

Anna Gierzkiewicz
Jagiellonian University

Oscillating orbits in the Sitnikov model

In a joint work with M. Capiński and P. Martín, we prove the existence of oscillatory orbits (*i.e.* orbits that go closer and closer to infinity but always return to a fixed bounded region) for the Sitnikov 3BP in the case of three equal masses ($m_1 = m$).

The proof relies on analyzing the stable and unstable invariant manifolds of infinity and their intersections. We construct orbits shadowing these invariant manifolds by the method of correctly aligned windows. The proof is computer assisted with the use of CAPD library for C++.

Dídac Gil Rams
Centre de Recerca Matemàtica (CRM)

Splitting of separatrices in generalized standard maps

We study transversal intersections between the invariant manifolds (stable and unstable) associated to an hyperbolic fixed point for a class of maps. These intersections are known as homoclinic orbits. The existence of these kind of orbits is one of the most celebrated methods to prove the existence of chaotic dynamics in a system. Indeed the Birkhoff-Smale theorem ensures that if there exist transversal intersections between the invariant manifolds of the same invariant object, the system is locally conjugate to a Smale horseshoe with infinite symbols.

The classical Melnikov theory is a first order perturbative theory that, in addition, can be used to measure the intersection angle between the invariant manifolds. However straightforwardly there are cases where the Melnikov function is exponentially small and the associated theory is not true. In these cases, to measure, for example, the intersection angle between the manifolds, becomes a difficult and technical task, since it is a beyond all orders phenomenon. This is the case of the problem we are considering.

We study the splitting of separatrices on generalized standard maps. This generalization includes the *standard map*, first studied by Lazutkin, the *perturbed McMillan map*, or the *Hénon map* among others.

More concretely, we are going to study the intersection of the invariant manifolds associated to a fixed point of the discrete dynamical system

$$\begin{cases} x^* = x + y + f(x,h), \\ y^* = y + f(x,h), \end{cases}$$

where h is a small parameter and f depends analytically on $|h| < h_0$, $|x| < \rho_0$, for some fixed $h_0, \rho_0 > 0$. We consider f to be of the form

$$f(x,h) = \sum_{k\geq 0} f_k(x) h^{k+2}, \quad f_k(x) = \begin{cases} \sum_{j=1}^{d_k} f_{k,j} x^j & \text{polynomial}, \\ \sum_{j=-d_k}^{d_k} f_{k,j} e^{ijx} & \text{trigonometric}, \end{cases}$$

with $f_{k,d_k} \neq 0$ for some k. In addition some extra condition on the exponents d_k are imposed.

We obtain an asymptotic formula for the *Lazutkin invariant*, value related to the area between two homoclinic points, and its first term depends on a Stokes constant that is generically different from zero. To do so, one of the techniques that we use is the *inner equation* related to our generalized standard maps.

The Stokes constant is an involved quantity that comes from the study of the inner equation. The second part of our work consist in giving a general algorithm, useful for the studied maps, to compute an interval containing the named constant (i.e., to see if it is different from zero) by means of a computer assisted proof in CAPD . Finally, we apply this algorithm to the Standard and Hénon maps in order to see that the intervals contain the values given by Gelfreich.

Joint work with Inmaculada Baldomà Barraca (UPC), Maciej Capiński (AGH) and Pau Martín de la Torre (UPC).

Joan Gimeno
Universitat de Barcelona

An Explicit Normal Form Computation on Discrete Systems

This talk presents a semi-analytical algorithm for computing normal forms in discrete dynamical systems, such as Poincaré maps. The method consists of two main components. First, assuming the existence of an invariant object, the algorithm computes a high-order local approximation in its neighborhood using jet transport techniques, which efficiently evaluate derivatives. Second, it performs a sequence of coordinate transformations—via composition and inversion operations—to simplify the local dynamics, retaining resonant and other meaningful terms essential for understanding the local behavior of the system.

The algorithm is general-purpose, requiring only smoothness assumptions, and remains robust under parameter variations. It enables the constructive computation of parametrized normal forms with respect to system parameters. In particular, in settings such as Hamiltonian systems with elliptic periodic orbits, the method naturally yields high-dimensional twist maps. Furthermore, a frequency recovery procedure is incorporated to explicitly compute both full and sliced invariant tori emanating from the elliptic periodic orbit.

To illustrate the practical power of this approach, I will apply it to the spatial Restricted Three-Body Problem (R3BP), demonstrating how it can uncover detailed aspects of orbital dynamics near the L4 point. The talk will include computational examples, implementation details, and a discussion of potential extensions and broader applications.

I welcome questions, suggestions, and discussions on adaptations or specific experimental setups that could benefit from this methodology.

This has been a joint work with À. Jorba, M. Jorba-Cuscó, and M. Zou.

Javier Gomez-Serrano
Brown University

Existence of analytic non-convex V-states

V-states are uniformly rotating vortex patches of the 2D Euler equation. The only known explicit examples are circles and ellipses: the rest of positive existence results use local or global bifurcation arguments and don't give any quantitative information of the solutions. In this talk I will prove the existence of solutions far from the perturbative regime, being able to extract nontrivial features of them and a precise quantitative description. The proof uses a combination of analysis and computer-assisted techniques.

Joint work with Gerard Castro-López.

Alex Haro
Universitat de Barcelona and CRM

Some translated tori theorems and applications

I will present some translated tori theorems and applications to the bifurcation theory of invariant tori and celestial mechanics.

David Hien
Technical University of Munich

Cycling Signatures: Identification of Cycling Motions and Their Transition

Recurrence is a fundamental characteristic of dynamical systems with complicated behavior. Understanding the inner structure of recurrence is challenging, especially if the system has many degrees of freedom and is subject to noise. The cycling signature is an algebraic topological notion for identifying and classifying elementary recurrent motions – called cycling – and the transitions between them. Statistics on these cycling motions can be computed from sampled trajectories (time series data), providing coarse global information on the structure of the recurrent behavior.

Formally, the cycling signature of a segment γ of a time series Γ is a map in homology $H_1(O(\gamma)) \to H_1(Y)$ where $H_1(O(\gamma))$ is the (persistent) homology of an offset filtration of γ and Y is a space which captures 'all possible cycling motions' that Γ might exhibit. A step in the computation of cycling signatures is computing a generating set of $H_1(O(\gamma))$. This can either be done via a standard persistence algorithm, or using an algorithm that computes zeroth persistent homology of the distance matrix of γ (regarded as a pixel image). The latter also yields additional structural information relevant to cycling transitions.

Joint work with Ulrich Bauer, Oliver Junge, and Konstantin Mischaikow.

Luc Jaulin
affiliationENSTA, Lab-STICCIntegral algebra for the validated integrati

Integral algebra for the validated integration of nonlinear differential inclusions

This talk presents an integral algebra and shows how it can be used to simulate a dynamical system with interval uncertainties. These uncertainties, can be either on the initial state vector, on the time-dependent inputs, or on the evolution function. Compared to other techniques used for the guaranteed integration of differential inclusion, the presented approach does not require the use of a fixed-point Picard operator. Several test-cases related to robotics are presented to illustrate the efficiency of the approach.

Piotr Kalita
Jagiellonian University

Stability of phase diagram for a gradient ODE with memory

We study the following gradient ODE

$$x'(t) = f(x(t)) \quad \text{where} \quad f \in C^2(\mathbb{R}^d;\mathbb{R}^d) \quad \text{and} \quad f = \nabla F. \tag{1}$$

Assuming that this equation has a global attractor, we consider the problem perturbed by the delay term

$$x'(t) = f(x(t)) + \varepsilon \int_{-\infty}^{t} M(t-s)x(s)\, ds, \tag{2}$$

where $M : [0,\infty) \to \mathbb{R}^{d\times d}$ is a time dependent matrix whose norm decays to zero exponentially as $t \to \infty$. The main result says, that if in the system (1) the equilibria are hyperbolic and their stable and unstable manifolds intersect transversally, then the structure of heteroclinic connections is preserved in the infinite dimensional system (2) if ε is small. The main tool is the Dafermos transform. We prove that in the perturbed problem the additional infinite dimensional variable that comes from the presence of the delay term after the Dafermos transform is always locally stable. This allows us to demonstrate that the local stable and unstable manifolds of all the equilibria of (2) when ε is small are C^1 close, in appropriate sense, to the local stable and unstable manifolds of (1). Consequently, the structure of transversal intersections is preserved in the perturbed system.

Jakub Kural
Jagiellonian University

Periodic orbits of ENSO delay differential equation model

El Niño–Southern Oscillation (ENSO) is a climate phenomenon of warming of oceanic waters of the western coast of equatorial South America, coupled with an oscillation in the surface air pressure between the eastern and western tropical Pacific. We investigate the ENSO delay differential equation model

$$x'(t) = -a\tanh(\kappa x(t-\tau)) + b\cos(2\pi t)$$

We rigorously prove the existence of a variety of periodic orbits for small values of κ. The proofs are computer-assisted with the use of the CAPD C++ library.

This is a joint work with dr Robert Szczelina and dr Anna Gierzkiewicz.

The research was funded by the program Excellence Initiative – Research University at the Jagiellonian University in Kraków.

Jean-Philippe Lessard
McGill University

Recent advances about the rigorous integration of parabolic PDEs via fully spectral Fourier-Chebyshev expansions

In this talk, we present a recent approach to rigorously solve initial value problems for semilinear parabolic partial differential equations (PDEs) using fully spectral Fourier-Chebyshev expansions. By reformulating the PDE as a system of nonlinear ordinary differential equations and leveraging Chebyshev series in time, we reduce the problem to a zero-finding task for Fourier-Chebyshev coefficients. A key theoretical contribution is the derivation of an explicit decay estimate for the inverse of the linear part of the PDE, enabling larger time steps. This allows the construction of an approximate inverse for the Fréchet derivative and the application of a Newton-Kantorovich theorem to establish solution existence within explicit error bounds. Building on prior work, our method is extended to more complex partial differential equations, including the 2D Navier-Stokes equations, for which we establish global existence of the solution of the IVP for a given nontrivial initial condition. This is joint work with Matthieu Cadiot (McGill).

Michał Lipiński
ISTA

Conley-Morse persistence barcode: homological signature of a combinatorial bifurcation

Bifurcation is one of the major topics in the theory of dynamical systems. It characterizes the nature of qualitative changes in parameterized dynamical systems. In this project, we study combinatorial bifurcations within the framework of combinatorial multivector field theory – a young but already well-established theory providing a combinatorial model for continuous-time dynamical systems. We introduce the Conley-Morse persistence barcode, a compact algebraic descriptor of combinatorial bifurcations. The barcode captures structural changes in a dynamical system at the level of Morse decompositions and provides a characterization of the nature of observed transitions in terms of the Conley index. The construction of Conley-Morse persistence barcode builds upon ideas from topological data analysis (TDA). Specifically, we consider a persistence module obtained from a zigzag filtration of topological pairs (formed by index pairs defining the Conley index) over a poset. Using gentle algebras, we prove that this module decomposes into simple intervals (bars) and compute them with algorithms from TDA known for processing zigzag filtrations.

The talk is based on: https://arxiv.org/abs/2504.17105

Filip Łanecki
Jagiellonian University

The Classifying Strength of the Leray Functor

Normal functors were introduced by Mrozek in order to define a Conley index for discrete dynamical systems. Later it was suggested that such functors may play a crucial role in defining connection matrices. In the class of all such functors one can find the universal one — called the Szymczak functor — which allows the index to retain the most detail. The Szymczak functor is strongly connected to the concept of shift equivalence, but has a fatal weakness — there is no known general method to decide whether two objects in its image are isomorphic; or even if any pair of non-isomorphic objects exists. On the other hand, there is the Leray functor, originally defined for the category of vector spaces, computable by means of simple linear algebra — very easily transferable to algorithms. We show that its construction can be generalised to abstract regular categories, while retaining computability, and that on finite objects of such categories it has the same classifying strength as the Szymczak functor. We also show how it can be adapted to work not only with maps, but also relations, we describe its classifying strength in the context of linear relations on modules.

Pau Martín
UPC

Chaotic phenomena around L4 in the RPC3BP beyond the Routh mass ratio

The Restricted Planar Circular 3 Body (RPC3BP)problem is a simplified model of the 3BP in which one of the masses is zero, the other two, the primaries, describe circular orbits and the massless body evolves in the plane of the bodies with mass. It is well known that, in suitable rotating coordinates, this model has five equilibria, three of them colinear with the primaries, of saddle-center type, and the other two, commonly known as L_4 and L_5, forming an equilateral triangle with the primaries. The linear type of these last equilibria depends on the mass ratio: when this is smaller than the Routh value, they are of center-center type, becoming complex saddles for larger values, after a Hamiltonian-Hopf bifurcation.

It is well known that the rescaled normal form of the Hamiltonian at L_4 is formally integrable, when expanded in powers of the difference of the mass ration and the Routh value. Hence, deciding if the invariant manifolds of L_4 intersect transversely is a beyond all orders problem. Transversal intersection of the manifolds would give rise to Smale horseshoes in the problem, with the well known appearance of chaotic motions and periodic orbits of arbitrarily high period.

In this work we obtain an explicit expression for the distance between the invariant manifolds of L_4. In particular, we prove that, if certain coefficient is different from 0, the invariant manifolds intersect transversely. This coefficient, often known as Stokes constant, depends on the full jet of the Hamiltonian. There is numerical evidence that it is not zero. To check that it is indeed different from 0 is work in progress.

Jakub Mazur
Faculty of Mathematics and Computer Science, JU

Effective algorithm for traversing all Depth Posets of a Lefschetz Complex

Given a filtration of a Lefschetz complex, the Depth Poset is the extension of a persistence diagram, that convey the information about the underlying filtration. As different filtrations can give different Depth Posets, we could extract useful information about the underlying complex by traversing all possible Depth Posets.

Here I present an algorithm that, given a Lefschetz Complex, allows for searching through all possible Depth Posets without looking through all filtrations. In particular, it can construct a filtration with Depth Poset of smallest possible height or biggest possible number of shallow pairs (minimal elements of a Depth Poset). Moreover, this allows us to find a Forman gradient vector field with minimal amount of critical cells or to check whether a complex admits a perfect Morse function.

Natalia McAlister
Monash University

A computer-assisted proof of the existence of blenders for a 3-dimensional Hénon-like family

In this project we develop a computer program to verify the existence of blenders for concrete examples. A blender is a hyperbolic set whose unstable manifold, when looking at certain intersections, seems to have a greater dimension than it actually does. This descriptive definition cannot be verified on a computer. The first step is then stating necessary conditions for establishing the existence of a blender in a computer-friendly way. Then, we develop an algorithm to verify said conditions. Using this algorithm, we prove the existence of blenders for a family of maps defined as a skew product over the Hénon map. This algorithm could be extended to other quadratic maps, and potentially to other more challenging examples.

Juan Miranda
Florida Atlantic University

Borel Transform: Validated Numerics and Applications

The Borel transform can be used to study functional equations whose solutions are C infinity but not analytic. Such equations describe invariant manifolds attached to parabolic fixed points and the solution of heat type equations. A fundamental challenge in working with the Borel transform is that it turns products into complex convolutions. This work aims to develop validated numerical methods for complex convolutions which can then be used in computer assisted proofs involving the Borel transform.

Jay Mireles James
Florida Atlantic University

A conjecture of Stromgren

In the 1920's, Stromgren made the following conjecture about the planar Lyapunov family at L1 in equal mass case of the Circular Restricted Three Body Problem (CRTBP): he conjectured that the family undergoes one symmetry breaking bifurcation, before accumulating to an asymptotic periodic orbit (what would in modern times be called a homoclinic orbits) at L4. In the 1970's, it was proven by Henrard that if a 2 freedom Hamiltonian vector field has a transverse homoclinic to a saddle focus equilibrium, then there is a tube of periodic orbits accumulating to this homoclinic. The tube can be parameterized by period, which goes to infinity as the tube accumulates the homoclinic. This deep result left open the question of Stromgren: wether this actually occurs in the CRTBP.

A few years agao, the local part of this question was settled by Maciej Capinski, Shane Kepley, and myself. Namely, we proved the existence of (several distinct) transverse homoclinic connections at L4 in the CRTBP. By the results of Hernard, each homoclinic has an associated tube of periodic orbits which accumulates to it. However, our argument does not produce the tube and hence leaves open the more global question of Stromgren. Namely, does the planar Lyapunov family accumulate – after one symmetry breaking bifurcation – to (what appears to be) the most symmetric ("shortest") homoclinic at L4 in the equal mass problem? I will discuss a computer assisted proof settling this conjecture in the affirmative. The argument relies heavily on the use of the Birkhoff normal form for the CRTBP at L4. To obtain requisite computer assisted error bounds on the tail of the normal form expansion, we utilize results of Delshams. This is joint work with Daniel Wilczak.

Marian Mrozek
Jagiellonian University

Combinatorial Links between Topology and Dynamics

Ties between topology and dynamics go back to the seminal work of Marston Morse in the the 1930s. Combinatorial links started with the discrete Morse theory by Robin Forman in the 1990s. I will present a recent observation that a filter in TDA is a combinatorial Morse function, inducing a hierarchy of combinatorial dynamical systems via a set of dependencies between birth-death pairs. This, in particular, enables the use of some topological dynamics tools in TDA.

Isaia Nisoli
UFRJ

Rigorous enclosure of the discrete spectrum for transfer operators

In this work, in collaboration with Blumenthal and Taylor-Crush, I present a generalization of a fundamental result, the Gerschgorin circle theorem, to obtain enclosures of the discrete spectrum of a transfer operator preserving a strong Banach space compactly embedded in a weak Banach space. The enclosures are obtained by rigorously bounding the weak resolvent norm of a finite rank approximation of the transfer operator. This result has important consequences, allowing us to understand the finer statistical properties of systems satisfying a Lasota-Yorke inequality, as uniformly expanding maps and systems with additive noise.

Michał Palczewski
Gdańsk University of Technology

Quiver Representations, Endomorphisms in Persistent Homology and the Conley Index Computation

We introduce a method for the computation of the homomorphism inuced in persistent homology by a continuous map, building on ideas from previous approaches [1, 2, 4]. The method combines a continuous function $f \colon X \to Y$ with the inclusion map $\iota \colon X \to Y$, allowing for the computation of the Conley index map, as discussed in [3]. Given topological spaces X and Y, along with sampled data $f|_S$ representing the restriction of f to a finite subset $S \subset X$, we propose a framework based on quiver representations for the computation of an endomorphism in persistent homology. We introduce the notion of approximate index pairs based on the sampled map, and we apply our framework to make an attempt to compute a persistence version of the Conley index.

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Aleksander Pasiut
AGH University of Krakow

Oscillatory orbits to collision and infinity in the planar circular restricted three body problem

In this talk we consider Earth-Moon planar circular restricted three body problem and we present the proof of the existence of orbits which oscillate between the collision and the infinity. We achieve our goal with the use of topological tools combined with rigorous interval computations. We use Levi-Civita regularization, McGehee regularization and validate that the dynamics in the regularized coordinates leads to a good topological alignment between various sets. We then perform shadowing arguments that this leads to the required dynamics in the original coordinates of the system.

Luke Peterson
University of Colorado Boulder

Computer-assisted Proofs for Subharmonic Melnikov Functions with Applications to the Earth-Moon-Particle System

Resonant periodic orbits are often used in astrodynamics and space mission design due to their predictable and repeatable behavior. For example, the nominal orbit of Lunar Gateway–the next-generation space station–is a 9:2 near-rectilinear halo orbit (NRHO) around Earth-Moon L_2, i.e., the station orbits the Moon 9 times in the time it takes the Sun to orbit the Earth-Moon frame 2 times. In the Earth-Moon system, the dynamical model used for preliminary mission design is the circular restricted 3-body problem (CR3BP)–an autonomous Hamiltonian system; however, there are significant perturbing effects, from the ellipticity of the Earth-Moon orbit to the gravitational influence of the Sun, that must be incorporated to ensure successful operation of the mission. Each of these effects can be modeled as periodic perturbations to the CR3BP–the Elliptic R3BP (ER3BP) or R4BP, respectively. In either case, most CR3BP periodic orbits persist as invariant 2-tori under certain non-degeneracy conditions prescribed by KAM theory; yet, resonant periodic orbits persist as resonant periodic orbits, often multiple orbits of the same resonance, as determined by subharmonic Melnikov theory. So, if there are multiple orbits with the same resonance in the perturbed (and more realistic) models, then which orbit is "the" 9:2 NRHO for Gateway? What is the relationship between the resonance of an orbit and the number of resonant orbits persisting into the perturbed system? Moreover, as these perturbations are not so small, are there bifurcations of these orbits, leading to a qualitative change in the dynamics?

In this work, we seek to prove the existence of a certain number of L_2 9:2 NRHOs in the Earth-Moon ER3BP. To do this, we break the problem down into several steps. First, we study lower-order resonant orbits in the same family–4:1 and 5:2. For each of the three resonant cases, we prove the existence of the periodic orbit in the CR3BP (the unperturbed problem) using computer-assisted techniques, e.g., the radii polynomial approach. Next, we show that the subharmonic Melnikov function, $\mathcal{M}$, of the ER3BP can be written in a particular form, enabling us to state and prove some pen-and-paper results for $\mathcal{M}$. The analytical results give insight into the relationship between the order of an orbit's resonance and the number of persisting periodic orbits (for small values of the perturbation), as well as a simplification of the evaluation of $\mathcal{M}$. As classical subharmonic Melnikov theory states rigorous results for lower-dimensional problems, we strive to state a subharmonic Melnikov theorem that has a classical flavor but for higher-dimensions; in other words, we develop a theorem relating the size of the neighborhood around the zeros of $\mathcal{M}$ with the size of perturbation magnitude and accuracy of prediction by $\mathcal{M}$. Then, applying the subharmonic Melnikov analysis to the three resonant orbit cases (4:1, 5:2, 9:2), we use numerical continuation to increase the perturbation magnitude to the "real" Earth-Moon ER3BP, wherein we re-work the radii polynomial approach to prove the existence of a particular number of resonant periodic orbits in the Earth-Moon ER3BP. This is a work-in-progress talk, and each of these steps may serve as a talk on their own. So, we work through only some of the details of the radii polynomial approach and subharmonic Melnikov theory, with the hope of communicating the flavor of our approach to solving an interesting problem in celestial mechanics and computational dynamics originating in a real-world engineering scenario (i.e., from astrodynamics, space mission design, and NASA). This is joint work with Gavin Brown (CU Boulder), J. D. Mireles-James (FAU), and Daniel Scheeres (CU Boulder).

Paweł Pilarczyk
Gdańsk Tech, Poland

Rigorous computation of expansion in one-dimensional dynamics

We introduce an effective algorithm and software for rigorous numerical computation of a lower bound for expansion in one-dimensional dynamics. The approach is based on interval arithmetic and efficient graph algorithms.

Specifically, let I be an interval, $\Delta \subset I$ be a subinterval (or a union of subintervals) of I and let $f\colon I \to I$ be a map which is C^1 on $I \setminus \Delta$. We say that f is *uniformly expanding outside* Δ if there are constants $C, \lambda > 0$ such that for any $x \in I$ and $n \geq 1$ with $f^i(x) \notin \Delta$ for all $i = 0, \dots, n-1$, we have

$$|(f^n)'(x)| \geq Ce^{\lambda n}. \tag{3}$$

Of particular interest to us is the case where Δ is a small neighbourhood of the critical points, in which case the derivative in $I \setminus \Delta$ can be very small and the expansivity condition (3) is far from obvious.

Given a function f, its domain I, and a set Δ satisfying the assumptions listed above, the purpose of our method is to compute explicit numbers C and λ, as large as possible, satisfying (3).

We provide a numerical technique building on and considerably improving similar techniques in the literature [1,2]. The rough idea is to split the set $I \setminus \Delta$ into a finite partition of intervals, and to use a directed graph with weighted edges to represent the map f together with estimates on its derivative on the partition elements. Using interval arithmetic to obtain rigorous bounds, and classical graph algorithms such as Karp's algorithm for the minimum cycle mean in a digraph, we obtain rigorous estimates for λ. Suitable partition of $I \setminus \Delta$, constructed in an iterative way, provides nearly optimal bounds, as opposed to the uniform partition previously used in [1,2].

This is joint work with S. Luzzatto and M. Palczewski.

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Gabriella Pinzari
Università di Padova

An application of non-quasi-periodic normal form theory to celestial mechanics

Normal form theory is a tool in hamiltonian mechanics, going back to N. N.Nekhorossev, that has been refined and clarified by many authors. In this talk we shall show that an extension to it to damped oscillations can be successfully applied to a spin-orbit model with friction. As a result, we shall prove that certain dumped oscillations do persist in the model for exponentially-long times. The physical menaning of this result allows to provide an interpretation of resonance trapping through the occurrence of friction. This is joint work with B. Scoppola and M. Veglianti.

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Philip Pita Forrier
Universitat de Barcelona

A study of the local behavior of the Taylor method for stiff ODEs

In this talk we study the behavior of the coefficients of the Taylor method for the solution of an ODE system

$$y' = f(t, y(t)), \quad y(t_0) = y_0, \quad t \in [t_0, T]$$

that is *stiff* in the sense of Söderlind, Jay and Calvo. We use the computational technique called *jet transport* to study how the use of a floating-point arithmetic induces an error in the Taylor coefficients related to the variational equations of the ODE. This error can exhibit dominant exponential growth in stiff problems, and it is naturally related to the stiffness indicator introduced by Söderlind *et al.* This behavior is illustrated by numerical experiments on classical stiff problems. In particular, we study the implications of this effect when high-order Taylor methods are used for extended precision computations.

Mateusz Przybylski
Jagiellonian University

A cohomology class as an invariant of shift equivalence for finite relations

Let X be a finite set. We can describe a relation R on X by a Boolean matrix, and conversely, a Boolean matrix yields a relation on X. In both settings, shift equivalence is a natural and important dynamical equivalence relation. It is strictly weaker than conjugacy, and corresponds roughly to "eventual conjugacy" ([2]). For matrices, one use is in the classification of shifts of finite type. For relations, it arises in defining a Conley index for computational approximations of dynamical systems. Classifications of shift equivalence are given in [3, 4, 5]. In this talk, I will present a complete invariant in terms of the period, the induced partial order on recurrent components, and the cohomology class of the relation on those components.

The result is as follows. There exists a least integer $p > 1$ such that there exists an integer $N > 0$ such that $R^{n+p} = R^n$ for all $n \geq N$; we call p the period of R. Additionally, R induces a partial order $R_{\leq}$ on the strongly connected components, determined by which components map to which. Finally, a choice of representatives of the strongly connected components induces a co-cycle $\xi \colon R_{\leq} \to \mathcal{L}_p$, where $\mathcal{L}_p$ is the collection of non-empty subsets of $\mathbb{Z}/p\mathbb{Z}$; we denote the cohomology class of ξ by $[\xi]$. Now, for a finite relation R, the triple $(R_{\leq}, p, [\xi])$ is a complete invariant of shift equivalence.

This is a joint work with Ethan Akin, Marian Mrozek and Jim Wiseman [1].

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Alessandro Pugliese
Univ. of Bari, Italy

Detection and proof of cusp bifurcations in multi-layer energy balance models

Cusp bifurcations occur generically in two-parameter dynamical systems. They are associated with the coexistence of multiple equilibria and can give rise to complex behaviors such as multistability and hysteresis. In this talk, I will present a recently developed computer-assisted method for detecting and rigorously proving the existence of cusp bifurcations. The approach combines a two-parameter continuation of the equilibrium manifold with a Poincaré index-based criterion to locate candidate bifurcations. These are then validated through a Newton–Kantorovich argument that yields a fully rigorous proof. I will illustrate the method by proving the existence of a cusp bifurcation - and the resulting bistability - in a class of multi-layer energy balance models.

This is joint work with Jean-Philippe Lessard (McGill University).

Simon Rohou
ENSTA, Lab-STICC

Reliable localization of an underwater robot based on tubes and topological degree

A mobile robot can be described as a dynamical system with bounded uncertainties. In this talk, we will first present an interval framework for enclosing the feasible trajectories in a non-linear and bounded-error context.

We will then introduce Simultaneous Localization And Mapping (SLAM), a concept that links the problem of estimating the state of a mobile robot to the mapping of an unknown environment in which the robot evolves. A SLAM algorithm exploits similarities in the environment to compute *loop closures* corresponding to places visited several times by the robot. Loop closures allow to propagate information from known states to states with large uncertainties. We will explain how the interval framework can deal with these inter-temporal measurements.

When the environment is wide and homogeneous, as is the case for the seabed, loop closures are difficult to guarantee due to ambiguities in the observations. To prove that a robot has completed a loop, whatever the uncertainties in its evolution, we use the notion of topological degree that originates in the field of differential topology. We will show that a verification tool based on the topological degree is an optimal method for proving robot loops. As these tools avoid bad convergences in SLAM algorithms, we will be able to apply them in difficult homogeneous environments with challenging scene recognitions. The presentation will be illustrated by real data from an Autonomous Underwater Robot performing a navigation at sea.

This is joint work with Luc Jaulin, Peter Franek and Michel Legris.

Damian Sadowski
Jagiellonian University

Combinatorial approach to sampled dynamics based on Gaussian process surrogate modeling

This is a joint work with Bogdan Batko. We propose a novel method combining combinatorial topological dynamics and Gaussian Process (GP) modeling, whereby we can characterize the global dynamics from a finite amount of data, with high confidence. More specifically, the data is used to construct a surrogate GP model. We then describe the dynamics using algebraic topological invariants inferred from a combinatorial multivector field built on the basis of the GP model. Our experiments show that relatively sparse data is sufficient to obtain a qualitative description of the underlying dynamics with high confidence. I will focus on presenting algorithms enabling our construction. It will be followed by experiments and their depictions.

Evelyn Sander
George Mason University

Computing Manifolds for Billiard Maps on Perturbed Elliptical Tables

Dynamical billiards consist of a particle on a two-dimensional table, bouncing elastically each time it hits the boundary. The successive bounce location plus bounce angle forms a two-dimensional iterated map, which was first studied by Birkhoff. On elliptical tables, the dynamics of billiard maps are completely integrable. The Birkhoff conjecture proposes that this is the only smooth convex table for which this is true. In this spirit, we present an implicit real analytic method for billiard maps on perturbed elliptical tables. This method allows us to compute stable and unstable manifolds using the parametrization method. While the results as yet are numerical only, our method is devised so it can in future be validated. This is joint work with Patrick Bishop, George Mason University and Jay Mireles James, Florida Atlantic University.

Robert Szczelina
Jagiellonian University

Investigating chaos in Delay Differential Equations

In recent years, a significant effort was made to study and prove the chaotic dynamics in Delay Differential Equations (DDEs), with particular attention given to canonical examples such as the Mackey–Glass equation. Several of the techniques requires rigorous numerical computations of a considerable scale. However, the infinite-dimensional nature of the DDEs presents some challenges, such as the complicated setup, finding good approximations, and carrying out the computer assisted proofs.

In this talk, I will present a pseudospectral approximation that reduces the DDE to a finite-dimensional system of Ordinary Differential Equations (ODEs) while preserving dynamical features of the original system. Due to the low-dimensionality of the resulting approximation, the computations are less demanding and can be done using known tools, such as CAPD rigorous ODE solvers, to efficiently validate some interesting dynamics that closely mirror those of the full DDE (such as periodic orbits in the chaotic regime of parameters).

I present some results and some problems that arise in this approximation. I believe those complications are intrinsic to the nature of DDEs and should persist for the original system. I will compare the results to the rigorous estimates obtained in computer-assisted proofs for the true DDE and I will argue that it should be possible in the future to extend those results to the original system using for example differential inclusions.

Acknowledgments: This is a joint work with A. Gierzkiewicz and J. Kural. The author would like to acknowledge the support of Polish National Science Center (NCN) grant no. 2023/49/B/ST6/02801 and Polish NAWA Bekker project no. BPN/BEK/2023/1/00170.

Warwick Tucker
Monash University

Generic Upper Bounds to the Cyclicity Problem and the Behavior of Lyapunov Constants

We study the center-focus problem for planar polynomial differential equations, which can be viewed as a local version of Hilbert's 16th problem. Based on the derivative of a Lyapunov function, used for verifying the existence of first integrals, we establish novel results regarding the cyclicity and the center-focus conditions. More precicely, under generic conditions, and for any degree, we find the cyclicity of the fixed point, and the size of the Bautin ideal generated by the Lyapunov constants.

This is joint work with Yovani Villanueva.

Mikael Vejdemo-Johansson
CUNY CSI / GC

Cohomology and Circular Coordinates

Jointly with Vin de Silva, and later joined by Dmitriy Morozov, my own career in topological data analysis started with a study of using persistent cohomology in homological dimension 1 to generate coordinate functions with values on a circle. This has become my most consistent topic of research through the years – both studying consequences (low-dimensional cohomology is much faster to compute than homology; and their barcodes are connected), applications (recognizing bifurcations in dynamical systems; generate robust mechanisms for intrinsically loopable mean motifs, with applications to motion capture and games programming) – as well as a source for extensions in the community (José Perea studying extensions of these techniques to principal G-bundles and much more complicated coordinate spaces). In this retrospective talk, I intend to give an overview of coordinatization in general, circular coordinates in particular, and some examples of the applications that we have found and studied.

Kelin Xia
Nanyang Technological University

Mathematical AI for Molecular Sciences

Artificial intelligence (AI) based Molecular Sciences have begun to gain momentum due to the great advancement in experimental data, computational power and learning models. However, a major issue that remains for all these AI-based learning models is the efficient molecular representations and featurization. Here we propose advanced mathematics-based molecular representations and featurization. Molecular structures and their interactions are represented by high-order topological and algebraic models (including Rips complex, Alpha complex, Neighborhood complex, Dowker complex, Hom-complex, Tor-algebra, Rhombille tiling, etc). Mathematical invariants (from persistent homology, Ricci curvature, persistent spectral, Analytic torsion, algebraic variety, etc) are used as molecular descriptors for learning models. Further, we develop geometric and topological deep learning models to systematically incorporate molecular high-order, multiscale, and periodic information, and use them for analysing molecular data from chemistry, biology, and materials.

Sushmita Yadav
IIT JODHPUR

Equivalence of Equicontinuity and Distality of Non -Autonomous Systems on Real line

This talk will focus on the topological dynamics of a non-autonomous dynamical system $(X, \mathbb{F})$, where X is a compact metric space and $\mathbb{F} = \{f_1, f_2, \ldots\}$ is a sequence of continuous surjective functions on X. In particular we will discuss equicontinuity and distality for non-autonomous systems on the interval. We will discuss the distality of the system using the enveloping cover $E_0(X) = \overline{\{\omega_k : k \in \mathbb{Z}\}}$ (where $\omega_n = f_n \circ f_{n-1} ... \circ f_1$). We use analytical tools to establish the equivalence of distality and equicontinuity for non-autonomous systems on the interval.